Fusion of Functional Brain Imaging Modalities using L-Norms Signal Reconstruction

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Abstract

EEG/fMRI fusion algorithms attempt to construct a spatiotemporal estimate of neuronal activity using data gathered from both fMRI and EEG modalities.

Recent advances in neuroimaging technologies like msMRI [1] do not reduce importance of the research attempts to devise plausible methodology to combine different functional brain imaging techniques as fMRI and EEG, which often bear coherent and complementary spatio-temporal information [2].

In the most common framework for posing this problem, which we follow, the result of such fusion constitutes a set of underlying activities from which the data observed in both modalities can be re-constructed with minimal error using adequate forward models for both modalities.

Although squared-error (l_2 norm) minimization leads to the best estimator in case of Gaussian data noise, absolute error (l_1 norm) minimization can lead to a more robust solution in the presence of outliers. This fact lead us to develop 2 somewhat complementary algorithms to obtain desirable solution for l_1 and l_2 error norms. To solve l_1 norm problem we present objective function in a form amendable to be minimized by efficient sparse LP-solver which gives us the unique solution. This method makes a number of simplifying assumptions which convert the EEG/fMRI integration problem into optimization of a convex function. l_2 norm solution is obtained through the steepest gradient-descent method.

We present results on both artificial data with realistic parametric structure and preliminary analysis of a difficult challenge benchmark for fusion methods more generally.

Introduction to Fusion

Fusion algorithms are employed in an attempt to construct a spatio-temporal estimate of neuronal activity using data gathered from multiple functional brain imaging modalities. Here, the estimate is built by placing a dipole in each voxel of the modality with highest spatial resolution, and estimating the time course of each dipole without constraining dipoles' orientation. The solution space thus consists of a matrix $\mathbf{S}$ of dimensionality $3N \times T$, which actually consists of 3 $N \times T$ matrices. Each such sub-matrix corresponds to the projection of the dipole to the specific axis [3]:

Modality	Matrix	Size	Estimate
EEG	$\mathbf{E}$	$M \times T$	$\hat{\mathbf{E}} = \mathbf{AS}$
fMRI	$\mathbf{F}$	$N \times U$	$\hat{\mathbf{F}} = \tilde{\mathbf{S}}\mathbf{B}$

Forward EEG/fMRI equations, where $\tilde{\mathbf{S}}$ is a $N \times T$ matrix which holds the dipoles strengths without orientation information and $\tilde{s}_{ij} = ||(s_{ij}^{(x)}, s_{ij}^{(y)}, s_{ij}^{(z)})||_2$

General Problem Formulation

The objective is to reconstruct $\hat{\mathbf{E}}$ and $\hat{\mathbf{F}}$ so as to minimize $\Delta_{\mathbf{E}} = \hat{\mathbf{E}} - \mathbf{E}$ and $\Delta_{\mathbf{F}} = \hat{\mathbf{F}} - \mathbf{F}$ using forward models with spatial filter $\mathbf{A}$ for EEG and temporal filter $\mathbf{B}$ for fMRI. Thus, introducing trade-off parameter α and regularization parameter β objective function to minimize can be presented as

$$C = \|\Delta_{\mathbf{E}}\|_l + \alpha\|\Delta_{\mathbf{F}}\|_l + \beta c(\mathbf{S}) \quad (1)$$

where l is the norm to characterize the goodness of fit and $c(\mathbf{S})$ can incorporate other constraints as the smoothness of the solution in time or in space, minimal norm requirement etc.

l_2 Error – Gradient Descent

Taking derivative on (1) in case of $l = 2$ leads to a simple gradient descent rule

$$\mathbf{S}_{\tau+1} = \mathbf{S}_{\tau} - \eta\frac{\partial E(\mathbf{S})}{\partial \mathbf{S}}, \text{where } \eta \text{ is a learning rate.} \quad (2)$$

$$\frac{\partial E(\mathbf{S})}{\partial \mathbf{S}} = \frac{\partial \Delta_{\mathbf{E}}(\mathbf{S})}{\partial \mathbf{S}} + \alpha\frac{\partial \Delta_{\mathbf{F}}(\mathbf{S})}{\partial \mathbf{S}} + \beta\frac{\partial c(\mathbf{S})}{\partial \mathbf{S}} \quad (3)$$

$$\frac{\partial \Delta_{\mathbf{E}}(\mathbf{S})}{\partial \mathbf{S}} = 2\mathbf{A}^T(\mathbf{E} - \mathbf{AS}), \frac{\partial \Delta_{\mathbf{F}}(\mathbf{S})}{\partial \mathbf{S}} = 2\mathbf{I}^{<3>} \star \left((\mathbf{F} - \tilde{\mathbf{S}}\mathbf{B})\mathbf{B}^T\right), \quad (4)$$

where $\star$ operation corresponds to element-wise product of two matrices.

l_1 Error Minimization - LP Minimization

Using defined abbreviations we formulate an initial LP problem as follows

$$\hat{\mathbf{E}} + \Delta_{\mathbf{E}} = \mathbf{E} \quad \text{Constraints} \quad (5)$$

$$\hat{\mathbf{F}} + \Delta_{\mathbf{F}} = \mathbf{F} \quad (6)$$

$$\tilde{s}_{ij} \geq 0 \quad \text{Region} \quad (7)$$

$$C = \|\Delta_{\mathbf{E}}\|_1 + \alpha\|\Delta_{\mathbf{F}}\|_1 \quad \text{Objective}, \quad (8)$$

where α is used to check different trade-offs between two modalities as well as to normalize their influence in the optimization criteria.

LP transformations

First we redefine each $|x|$, which are present in computation of C (8) and $\tilde{s}_{ij}$ (10), in a form suitable for LP through usage of slack variables x^+ and x^-. This transformation leads to a side effect - minimization of the sum of absolute values of $|s_{ij}|$.

Secondly we approximate vector norm $e = ||\mathbf{m}||$ to be used within an LP framework. Our solution is to note that the $\min(\cdot,\cdot)$ and modulus $|\cdot|$ functions can be used freely in a LP and then reduced to canonical form using standard transformations. Let $\{\mathbf{R}_i\}$ be a set of rotation matrices. To approximate $||\mathbf{m}||$ we let

$$e_i = ||\mathbf{R}_i\mathbf{m}||_1 \qquad e = \min_i e_i \quad (9)$$

where $||\cdot||_1$ denotes the l_1 norm. These can simply be added to the linear programming problem, enforcing the relation $e \approx ||\mathbf{m}||$.

Final LP form

We encode the definition of $\tilde{\mathbf{S}}$ into LP constraints matrix through presenting it as

$$\tilde{\mathbf{S}} = l(|\mathbf{S}_x|, |\mathbf{S}_y|, |\mathbf{S}_z|), \quad (10)$$

where $l(\ldots)$ is a linear formulation to approximate l_2 norm.

To represent (5) and (6) in a form suitable for LP we use Kronecker product $\otimes$ obtaining the constraints and the objective function in an extended canonical form for LPs,

$$(\mathbf{A} \otimes \mathbf{I}_T)\overline{\mathbf{S}} + \Delta_{\overline{\mathbf{E}}} = \overline{\mathbf{E}} \quad (11)$$

$$(\mathbf{I}_T \otimes \mathbf{B}^T)\overline{\tilde{\mathbf{S}}} + \Delta_{\overline{\mathbf{F}}} = \overline{\mathbf{F}} \quad (12)$$

$$\overline{\tilde{\mathbf{S}}} - l(|\mathbf{S}_x|, |\mathbf{S}_y|, |\mathbf{S}_z|) = 0 \quad (13)$$

$$\overline{\tilde{\mathbf{S}}} \geq 0 \quad (14)$$

$$C = ||\Delta_{\mathbf{E}}||_1 + \alpha||\Delta_{\mathbf{F}}||_1 + \beta||S||_1 \quad , \quad (15)$$

where $\mathbf{I}_Z$ is the identity matrix of size $Z \times Z$.

Simulation Data

To check the method artificial data was created. Brain volume is simulated as a half-sphere with 9 voxels in diameter, which gives us 132 voxels total to be considered. Simple single sphere model was used to generate gain matrix for EEG 11 sensors distributed across the half-sphere surface. We've generated random activation map $\mathbf{S}$ consisting of 5 voxels firing within 600ms interval after t_0=2 sec from the beginning of the timecourse (1 voxel per each 0ms, 200ms, 400ms and 2 voxels at 600ms after t_0) with the same amplitude but in different locations and with arbitrary orientation. Using this map clean EEG and fMRI were constructed through the forward equations. EEG was sampled at 10sps and fMRI at 1sps, so EEG time resolution in the experiment was 10 times higher than slow fMRI.

Additive noise was used to corrupt EEG signal: Gaussian noise with SNR=-5dB, which due to sparsity constituted equivalently %RMS≈23%, where

$$\%RMS = \frac{\sigma_{noise}}{max(x)} \times 100\% \quad (16)$$

for clean signal x. SNR for EEG was fixed across all experiments. An fMRI signal was also corrupted by additive Gaussian noise with variable SNR to do some noise sensitivity analysis (for fMRI SNR 2dB ≈ 35% %RMS).

Data Conditioning

Before analysis, both data sets (EEG E and fMRI F) and corresponding matrices (A and B) were normalized by estimated noise standard deviation in order to properly scale error terms as well as to remove difference between units of EEG and fMRI. Then weights in error terms were used to remove dimensionality effect by assigning $\alpha = MT/NU$.

As multiple tests with different SNR levels have shown, best estimates of activations were achieved when $\beta = \frac{1}{3NT\sigma_S}$, where σ_S is standard deviation of simulated activation map. It can't be known for real data but we're speculating here by using artificial data. Robust method to estimate β is the next goal to achieve in future research.

Results

Obtained solutions for fMRI SNR > 3 dB returned all 5 original activations as 5 highest obtained activations for duration of the experiment with ≈ 50% of energy spread through the rest of the volume. Lowering SNR down to 1dB lead to a stable detection of 3 out of 5 activations.

Real Data

The Finger Sequence Benchmark

Most benchmarks for modality fusion often do not rule out solutions using only one modality. In order to create a true test of any fusion algorithm we propose to recover a finger tapping sequence in the M1 Hand region around central sulcus. To the extent that digit somatopy can be spatially resolved by statistical methods [4] or various classifiers, the problem still poses a severe challenge for modalities such as BOLD or EEG alone. Recovering finger tapping sequences in faster time resolutions (we used 12 sec, 4 sec and 1 sec) must require more cross signal fusion exploitation from temporally and spatially rich signals. Solutions to this type of problem are nontrivial for any kind of fusion approach.

Results

We report initial attempts using suggested benchmark where subjects finger tapped in fixed sequences at different time resolutions between digits. Digits were taken against a REST block and in contrast to all pair wise combinations (10). We preconditioned EEG using ICA and found components that were associated with design frequencies so to eliminate unrelated activity from the fusion process.

To label voxel somatopy we used neural network classifiers, which are known to generate stable topographic mappings in cases of non-linear decision surfaces.

Discussion and Future Work

In the future we plan to apply proposed methods to the challenge benchmark and show how signal fusion can produce reliable estimates of the rank order sequence of digits in time.

NN Localization vs GLM: Neuroimaging data patterns can often require models that are sensitive to higher complexity then linear frameworks such as GLM can provide. Neural networks can be used to identify voxel function even when prospective decision surfaces are graded and nonlinear [5]. Networks that are overparameterized also provide for simple filtering as well as smoothing of target clustered inputs. Most useful in the present hand somatopy problem is their ability to separate near overlapping non-convex regions and to smooth falloff from multiple maxima.

Modalities Tradeoff: The presence of the weight factors α and β and necessity to scale both modalities to find correspondence between their signal levels make proposed method less attractive while no efficient and robust method for such parameters estimation is suggested. Iterative re-estimation can be used as the one of possible approaches to accommodate useful variance of both modalities during fusion process [6].

fMRI Resolution: Although fMRI spatial resolution is quite high in comparison to localization given by EEG/MEG inverse solutions, agglomerating haemodynamic nature of the BOLD signal makes it quite hard to discriminate activations in the neighboring voxels. Increased spatial resolution due to selective slice acquisition should improve the results in case when ROI is well localized, such as in the suggested benchmark experiment.

BOLD Variability: HRF filters must be estimated for each voxel to account for variability of BOLD dynamics within the brain.

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